- Definition: A function is something that sends inputs to outputs, under one special rule:
  - for each input, there is exactly one output.

  (That is, input uniquely determines the output.)
- Input = "independent variable".
- Output = "dependent variable".

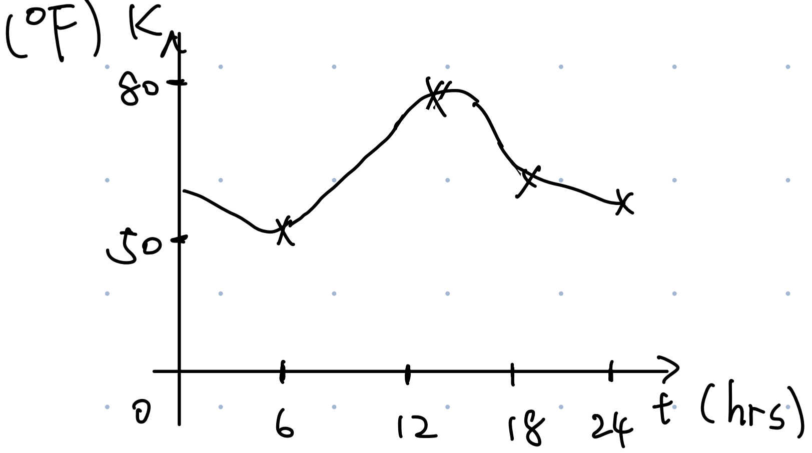
- Definition: The domain of a function is the set of all possible inputs.
- Definition: The range of a function is the set of all possible outputs.

- Example: A blender is a function that sends:

Input		Output
Apple	$\longmapsto$	Apple juice
Orange	$\longmapsto$	Orange juice
Vegetable	$\longmapsto$	vegetable juice

- Domain of "blender" = {Apple, Orange, Vegetable},
- Range of "blender" = {Apple juice, Orange j., v.j}.

- Example: The average temperature in the park at given time of the day is a function: time $t \longrightarrow$ temperature K



"Height = value".

Input	$\longrightarrow$	Output
6		50
12		80
18		65
24		60
9		75

- We write $K(6) = 50$, $K(12) = 80$ notationally.
- Domain $= [0, 24] =$ "all numbers between 0 and 24",
- Range $= [50, 80] =$ "all numbers between 50 and 80",

- What's not a function?

$$\begin{tikzcd} a \arrow[r, mapsto] \arrow[dr, mapsto] & 2 \\ b & 3 \end{tikzcd} \qquad \begin{tikzcd} a \arrow[r, mapsto] & 2 \\ b \arrow[ur, mapsto] & 3 \end{tikzcd}$$

not function ✗ $\qquad$ function ✓

- In this course, we focus mostly on "mathematical" functions, that are functions with all inputs and all outputs are numbers.

- <u>Example</u>: (scalar multiplication)

  Function f sends every number to its double.

  $f(1) = 2$, $f(2) = 4$, $f(7) = 14$, $f(1.3) = 2.6$

  $f(-5) = -10$. Domain$(f) = (-\infty, \infty)$, Range$(f) = (-\infty, \infty)$.

  any number

- We write $f(x) = 2x$ in this case. x is any input, and the output of f is $2x$. We call $f(x)$ the <u>value</u> of f at x.

- <u>Example</u>: (square function) $f(x) = x^2$.

  $f(1) = 1$, $f(2) = 4$, $f(1.4) = 1.96$, $f(-3) = 9$.

- Domain $= (-\infty, \infty)$, Range $= [0, \infty)$ = "any number ≥ 0".

- <u>Example</u>: (Square root function), $f(x) = \sqrt{x}$ = (Number ≥ 0 that number$^2 = x$)

  $f(1) = 1$, $f(4) = 2$, $f(1.3) = 1.1401\ldots$

  $f(-1)$ = not defined.

- Domain $= [0, \infty)$, Range $= [0, \infty)$.

- For math functions, neither domain nor range need to be everything!

- **Defn**: We say two functions f, g are equal, if
  1. f, g have the same domain.
  2. $f(x) = g(x)$ for every x in the domain.

- Common tricks to find the domain:
  1. Can't divide by zero.
  2. Can't take the square root of a negative number.

Example: Find the domain of

$$f(x) = \frac{\sqrt{x-3}}{x-4}.$$

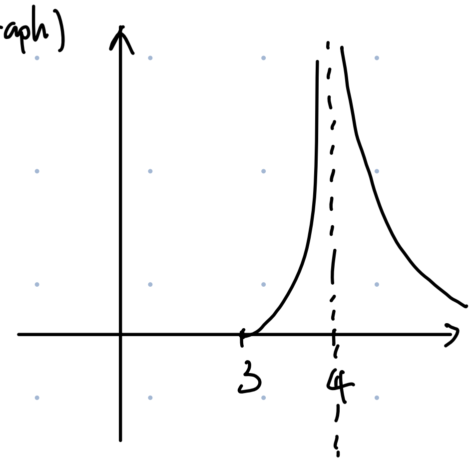
Solution: • Denominator can't be zero: $x-4 \neq 0$, $x \neq 4$.

- Can't take the square root of negative number:

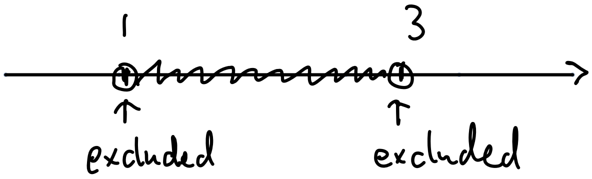
  $x-3 > 0$, $x > 3$.

- So the domain of $f = \{x > 3 : x \neq 4\} = (3,4) \cup (4, \infty)$.

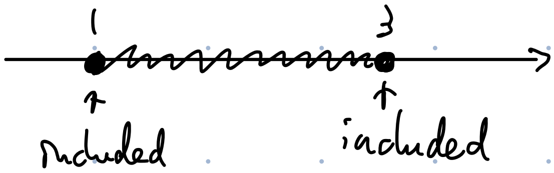
- Plot (graph)



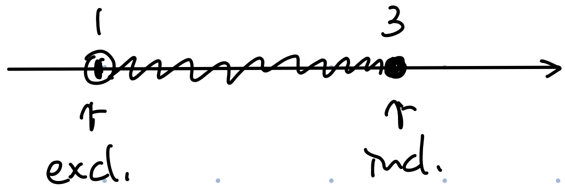
- Interval notation: "(" = exluded, "]" = included



$(1,3) = \{1 < x < 3\}$.



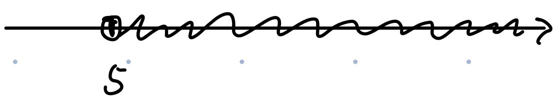
$[1,3] = \{1 \le x \le 3\}$



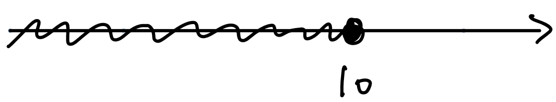
$(1,3] = \{1 < x \le 3\}$.

- $x \in [1,3]$ reads "x is in $[1,3]$", means $1 \le x \le 3$, or say x is a point between 1 and 3 with endpoints included.

- Some special integrals: • ∞ = infinity (larger than anything).



$(5,\infty) = \{x > 5\}$



$(-\infty, 10] = \{x \le 10\}$.

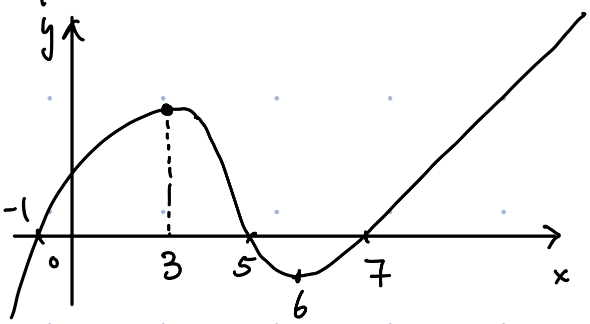
$(-\infty, \infty) = \{$any (real) number$\}$.
= "everything".

- Always use "(/)" with infinity in this class.

Change of Function:

- **Def'n**: f is positive / negative at a if $f(a)$ is positive/negative.
- **Def'n**: f is increasing / decreasing on an interval I (for example, $[a,b]$), if $x_2 > x_1$ implies $f(x_2) > f(x_1)$ / $f(x_2) < f(x_1)$.
- That is, the output of f increases as the input increases.

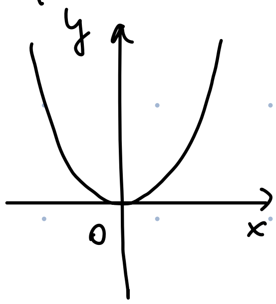
Example:



- f is positive on $(-1,5)$ and $(7,\infty)$
- f is negative on $(-\infty,-1)$ and $(5,7)$
- f is increasing on $(-\infty,3)$ and $(6,\infty)$
- f is decreasing on $(3,6)$.

- $\frown$ = concave down, $\cup$ = concave down

Example: $f(x) = x^2$:



- positive on $(-\infty, \infty)$
- increasing on $(0,\infty)$
- decreasing on $(-\infty, 0)$
- "concave up"

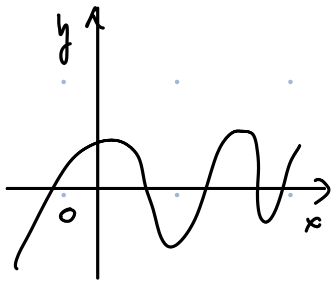
- Informal definition of "continuous".
- We say f is continuous at a, if

"x close to a" implies "$f(x)$ close to $f(a)$".

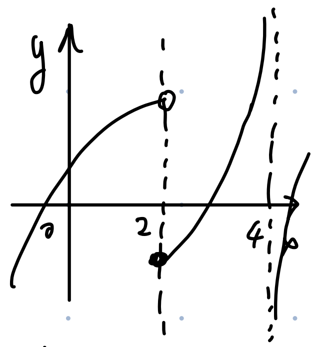
Visually, there is no gap nor hole in the graph of f at a.

- f is <u>discontinuous</u> at a if there is a gap or hole at a. "bad points".
- discontinuous = not continuous.

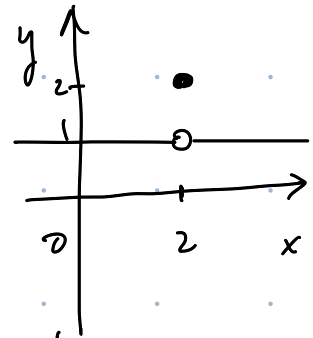
<u>Example</u>:



continuous everywhere



discontinuous at 2, 4.



$f(x) = \begin{cases} 1, & x \neq 2 \\ 2, & x = 2 \end{cases}$

discontinuous at 2

- Continuous functions are "good" functions: it changes gradually in a tracable way.