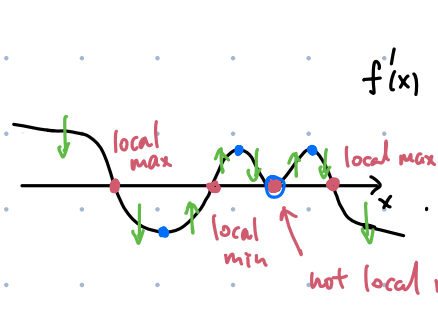


Ex. Given the graph of $f'(x)$, identify the following of $f(x)$:



- ① Stationary points.
- ② Inflection points.
- ③ Where f is concave up/down.
- ④ Local max/min.

Ex. The accumulative profit function of a company is given by

$$P(t) = \frac{1}{3}t^3 - 6t^2 + 11t + 30, \quad t \in [0, 18] \text{ (months)}$$

- (I) Find all stationary pts of $P(t)$. Identify local max/min and interpret them.
- (II) Find the point of inflection. Interpret.
- (III) Find the time t where $\begin{cases} \text{the profit is maximized.} \\ \text{the loss} \end{cases}$

Sol.ⁿ (I) Stationary pts: $P'(t) = 0$

$$P'(t) = t^2 - 12t + 11 = (t-1)(t-11) = 0,$$

$$\textcircled{1} t = 1, \quad \textcircled{2} t = 11.$$

$$P''(t) = 2t - 12$$

$$\textcircled{1} t = 1, \quad P''(1) = 2 - 12 = -10 < 0. \quad \boxed{N} \text{ Second der test } \Rightarrow \text{local max.}$$

Interpretation: the profit reaches a new high locally near $t = 1$ month, and then began to decrease.

$$\textcircled{2} t = 11, \quad P''(11) = 2(11) - 12 = 10 > 0. \quad \boxed{U} \text{ Second der test } \Rightarrow \text{local min.}$$

Interpretation: the profit reaches a new low local near $t = 11$ month, and then began to increase.

(II). Point of inflection: $P''(t) = 0$

$$P''(t) = 2t - 12 = 0 \Rightarrow t = 6.$$

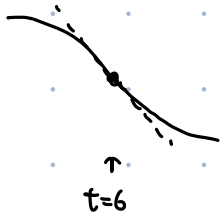
$$\text{Here } P'(6) = -25 < 0 \Rightarrow P \text{ decreasing.}$$

$$t < 6: P''(t) < 0 \Rightarrow P'(t) \text{ decreases on the left of } t = 6, \quad P \text{ concave down.}$$

$$t > 6: P''(t) > 0 \Rightarrow P'(t) \text{ increases on the right of } t = 6, \quad P \text{ concave up.}$$

• Point of inflection: point where the function changes concavity.

Interpretation: the profit was decreasing faster and faster before $t=6$, but then decreases slower and slower after $t=6$. (the company is losing less and less money after $t=6$.)



(II). Critical points: ① $t=1$, ② $t=11$, ③ $t=0$, ④ $t=18$.

Evaluation: $P(t) = \frac{1}{3}t^3 - 6t^2 + 11t + 30$.

① $t=1$, $P(1) = \frac{1}{3} - 6 + 11 + 30 = 35 + \frac{1}{3} = \frac{106}{3} \approx 35.3$

② $t=11$, $P(11) = \frac{1}{3}(11^3) - 6(11)^2 + 11(11) + 30 = -\frac{394}{3} \approx -131.3 \leftarrow \text{min.}$

③ $t=0$, $P(0) = 30$

④ $t=18$, $P(18) = 228 \leftarrow \text{max}$

• Max profit: $P(18) = 228$.

• Min profit: $P(11) = -131.3$

