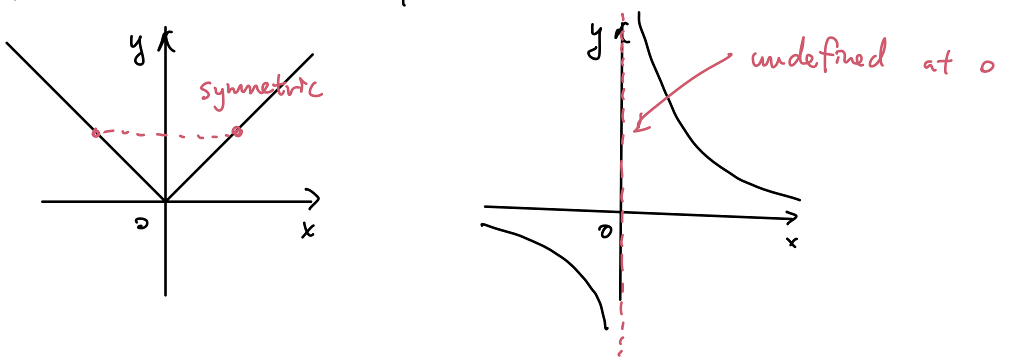
(*) Useful functions (S-2.2)

- Identity function : $f(x) = x$.
- Squaring function : $f(x) = x^2$
- k-th power function: $f(x) = x^k$, for integer $k = 1, 2, \cdots$.
- Absolute value function: $f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$.
- Reciprical function: $f(x) = 1/x$, if $x \neq 0$.
- Graph of absolute & reciprical function:

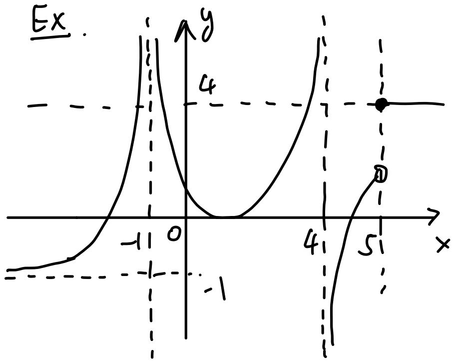


- We say $f(x)$ has a <u>vertical asymptote</u> at $x = a$, if

  ① $f(x)$ is not defined at $x = a$,

  and ② $f(x)$ gets exceedingly large/small as x gets closer to a.
- We say $f(x)$ has a <u>horizontal asymptote</u> at $y = c$, if $f(x)$ gets closer and closer to c as x increases/decreases indefinitely.

Ex.



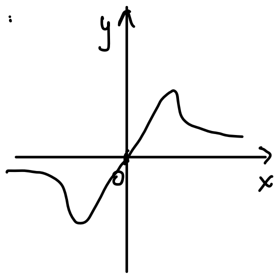
The function $f(x)$ has vertical asymptotes at $x=-1$ and $x=4$, but not at $x=5$.

$f(x)$ has horizontal asymptotes at $y=-1$ and 4.

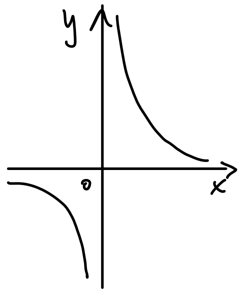
Def^{n}. • We say $f(x)$ is <u>even</u> if $f(-x)=f(x)$ for all x in the domain.

• We say $f(x)$ is <u>odd</u> if $f(-x)=-f(x)$ for all x in the domain.

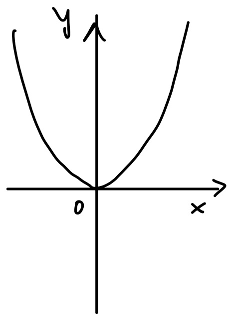
Ex.



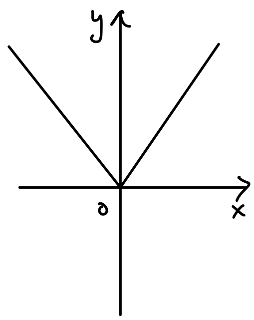
odd



odd

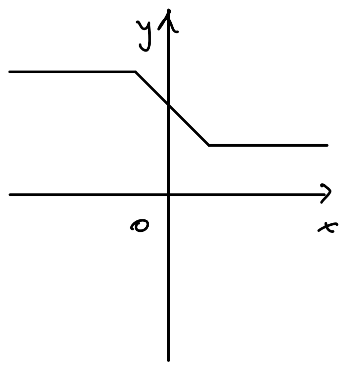


even



even

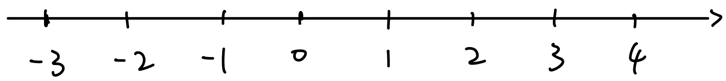
- even function = symmetric about y-axis
- odd function = symmetric about 0.
- There are many functions neither even nor odd.



NOT even, NOT odd.

- $|a-b|$ = distance between a and b. ≥ 0

Ex



$|3-1|$ = distance between 1 and 3 $= 2$

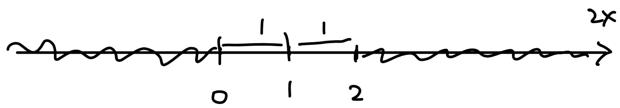
$|1-4|$ = ———" —— 1 and 4 $= 3$.

$|3+3| = |3-(-3)|$ = distance ... 3 and -3 $= 6$.

Ex. Solve $|2x-1| > 1$.

Sol^n. ① Geometric approach:

$|2x-1|$ = distance between $2x$ and 1 > 1.



$2x$ must be in the shaded region,

Thus either $2x > 2$ or $2x < 0$,

thus is either $\underline{x > 1 \text{ or } x < 0}$.

② Analytic approach:

- Case 1: $2x-1 \geq 0$.

We have $2x-1 \geq 0$ and $2x-1 > 1$
$$\left.\begin{array}{l} x \geq \frac{1}{2} \text{ and } x > 1 \end{array}\right\} x > 1.$$

- Case 2: $2x-1 < 0$.

We have $2x-1 < 0$ and $-(2x-1) > 1$
$$\left.\begin{array}{l} x \leq \frac{1}{2} \text{ and } x < 0 \end{array}\right\} x < 0.$$

The final answer is then $\underline{x>1 \text{ or } x<0}$. $\boxed{\text{Ex}}$

⊛ Average rate of change (§2.3)

<u>Q</u>: How do we measure how fast things are changing?

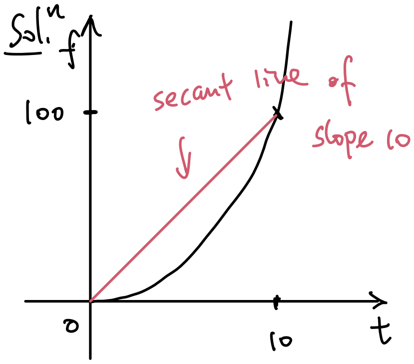
<u>A</u>: One reasonable answer is to divide the change in quantities by the change in time.

<u>Defⁿ</u>. The <u>average rate of change</u> of a function $y=f(x)$ over the interval $[a,b]$ is

$$\frac{f(b)-f(a)}{b-a} \underset{\text{denoted}}{=\!=} \frac{\Delta f}{\Delta x} = \frac{\Delta y}{\Delta x}.$$

<u>Ex</u>. Let a rocket accerating into the space be displaced at $f(t)=t^2$ at each time $t \geq 0$. Find the average rate of change of the displacement $f^{(m)}$ over time interval $t \in [0,10]$.

<u>Solⁿ</u>.



- Average rate of change
$$= \frac{f(10)-f(0)}{10-0} = \frac{100-0}{10} = 10 \ (m/s)$$
- Geometrically, the average rate of change = the slope of <u>secant line</u> connecting the endpoints.
- Physically, the average rate of change in this case is the average speed of the rocket over the first ten seconds.

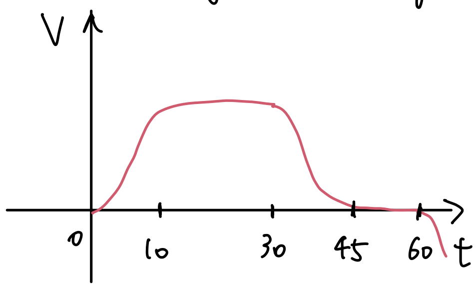
Ex. In the previous example, what is the average rate of change from $t=10$ to 20?

- Avg. rate of change $= \frac{\Delta f}{\Delta t} = \frac{f(20)-f(10)}{20-10} = \frac{20^2-10^2}{10} = \frac{300}{10} = 30$ (m/s).
- $30 > 10$: we are accelerating!
- $f(20) = 20^2 = 400$, $f(10) = 10^2 = 100$.

Δf = increase $= 400 - 100 = 300$.

The displacement/height of the rocket inreased by $\frac{\Delta f}{f(10)} = \frac{300}{100} = 300\%$ in 10 seconds after $t=10$.

Ex. The (signed) velocity V of a swimmer with respect to time t is



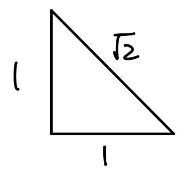
- They accerated from $t=0$ to 10,
- Then keep the same speed from $t=10$ to 30
- They began to slow down from $t=30$ to 45.
- Then took a rest/not moving from 45 to 60.
- And start to swim backward from 60 onwards.

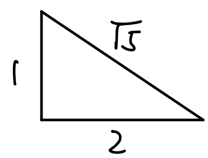
(*) More on numbers (§2.4)

Def^n. • Integers are numbers of the form $0, 1, 2, 3, \cdots$ or $-1, -2, -3, \cdots$.

- Rational numbers are numbers that can be written as $\frac{\text{integer}}{\text{integer}}$, for example $\frac{1}{3}$, $-\frac{7}{9}$, $0.97 = \frac{97}{100}$, or $7 = \frac{7}{1}$.
- Irrational numbers are those cannot be written as the ratio of integers.
- Real numbers are the set of irrationals and rationals.

<u>Example</u> of irrational number





$\sqrt{2}$ and $\sqrt{5}$ are irrationals.