⊛ Linear functions (§4.2).

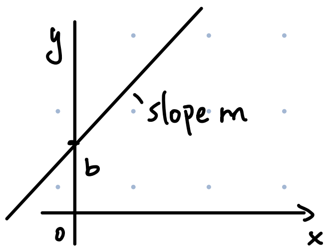
- Every line can be writen in either form:

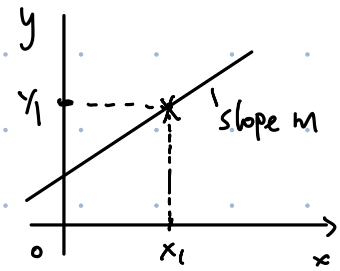
① Slope - intercept:

$$y = mx + b.$$

② Point - slope:

$$y - y_1 = m(x - x_1)$$





- Slope m = rate of change $= \frac{\Delta y}{\Delta x} = \frac{y - y_1}{x - x_1}$.
- We call functions of form $y = mx + b$, linear functions.
- They are functions that changes at the same rate all the time.

Ex: Bob is running at a constant speed. Bob is 40 meters far at time $t = 10$ s, and 80 meters far at time $t = 30$ s.

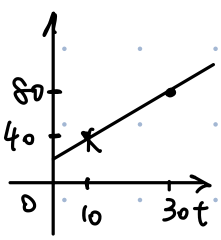
① Find the linear function $f(t)$ that describe how far Bob is at time t (s).

② How far was Bob at time 0?

③ What is the time when Bob finishes 100 meters?

④ Find the slope, Juteprpet the slope.

Sol^n. ① Slope $m = \frac{\Delta y}{\Delta x} = \frac{f(30) - f(10)}{30 - 10} = \frac{80 - 40}{20} = 2$ (m/s).



- Point - slope: $y - 40 = 2(t - 10)$, $y = 2t - 20 + 40 = 2t + 20$.
- The linear function is $f(t) = 2t + 20$

② $f(0) = 2 \times 0 + 20 = 20$ (m)

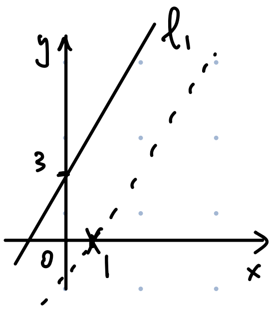
③ Let $f(t) = 100 = 2t + 20$, $t = 40$ (s).

④ Slope $= 2$ (m/s) = rate of change in distame run with respect to time

= velocity.

- Horizontal lines are of form $y = b$. $(m=0)$
- Vertical lines are of form $x = c$ for constant c.
- Two lines $y = m_1 x + b_1$, $y = m_2 x + b_2$ are $\begin{cases} \text{parallel} & \text{if } m_1 = m_2 \\ \text{perpendicular} & \text{if } m_1 \cdot m_2 = -1. \end{cases}$

Ex. Find the line that is parallel to $y = 4x+3$, and goes through $(1,0)$.



Sol.ⁿ Slope of $l_1 = 4$.

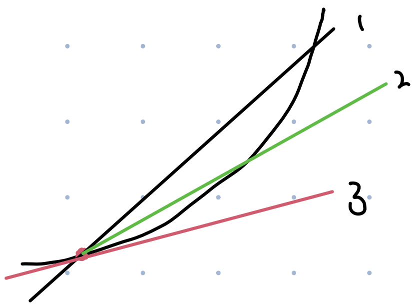
Parallel lines have the same slope.

New line: $y = 4x + b$.

$(1,0)$ on the line: $1 = 4 \times 0 + b$, $b = 1$.

New line: $y = 4x + 1$. $\boxed{\Sigma}$

- Why linear functions? We need them to approximate functions locally.



The slope of best approximation line = "derivative" (red line).

- Many real-world phenomenons are only locally like a line.

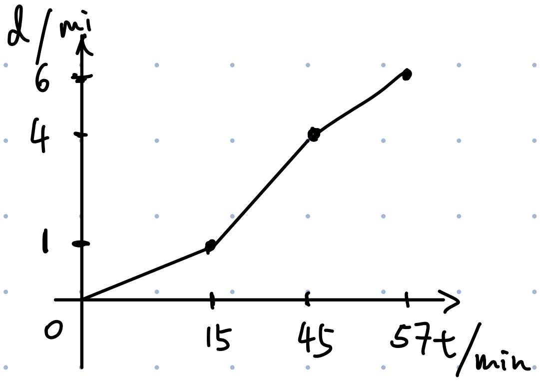
Example: Bob runs on a treadmill. They started at 4 mph for 15 min, $(\frac{1}{15}\ mi/min)$ then 30 min at 6 mph, $(\frac{1}{10}\ mi/min)$ and finish the run with a dash at 10 mph, $(\frac{1}{6}\ mi/min)$ for 12 min.

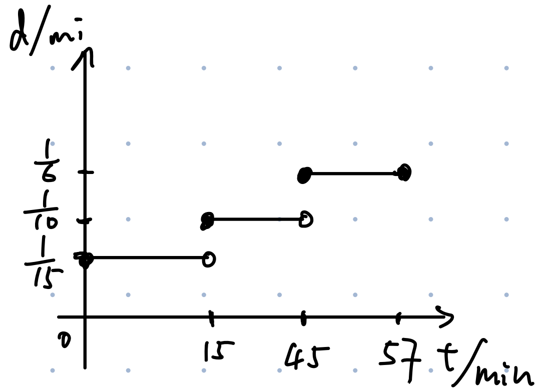
① Plot the displacement versus time graph.

② Plot the velocity versus time graph.

③ Find the average change of displacement in these 57 mins.

Sol.ⁿ ① Displacement $d(t) = \begin{cases} \frac{1}{15}t, & t \in [0,15) \\ 1 + \frac{1}{10}(t-15), & t \in [15, 45) \\ 4 + \frac{1}{6}(t-21), & t \in [45, 57]. \end{cases}$





② Velocity $v(t) = \begin{cases} \frac{1}{15}, & t \in [0, 15) \\ \frac{1}{10}, & t \in [15, 45) \\ \frac{1}{6}, & t \in [45, 57] \end{cases}$.

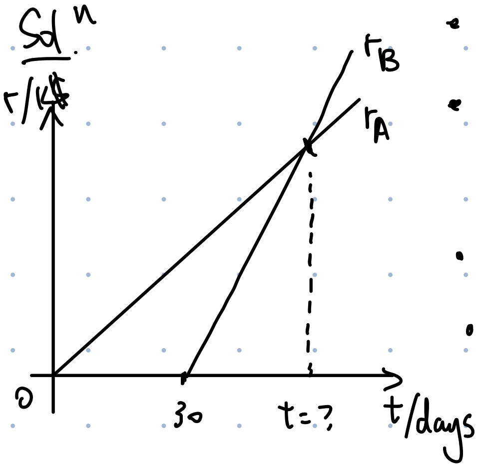
③ Average rate of change of $d(t)$ over $t \in [0, 57]$

$$= \frac{\Delta d}{\Delta t} = \frac{d(57) - d(0)}{57 - 0} = \frac{6}{57} = \frac{2}{19} \text{ mi/min} \approx 6.32 \text{ mph}.$$

- This is the average velocity of the run.

Ex. A company sells goods A from day 0 and makes revenue at 1,000\$ per day. 30 days later, they begins to sell goods B at 1,500\$ per day. Find after which day, the total revenue from B exceeds that from A.

Sol^n



- $r_A(t) = t$.
- $r_B(t) = \begin{cases} 0, & t \in [0, 30) \\ 1.5(t-30), & t \geq 30. \end{cases}$
- Want to solve $r_B(t) = r_A(t)$.
- From the graph, the intersection point happens after $t=30$.
- $r_A = t = 1.5(t-30) = r_B$.

  $45 = 0.5t$, $t = 90$.
- After 90 days, the total revenue from B exceeds that from A.