

* Transformation of functions (S3.4)

Motivation: given a function $f(x)$, want to modify it to get other functions.

Basic operations: **I** Shift, **II** Scale (stretch/shrink) **III** flip.

I Shifts: c is real.

- $f(x) + c$: **up**-shift by c units. (If $c < 0$, it indicates a down-shift)
- $f(x+c)$: **right**-shift by $-c$ units. (If $c < 0$, it indicates a left-shift)

II Scale: $k > 0$.

- $kf(x)$: scale vertically by ratio k .
- $f(kx)$: scale horizontally by ratio $1/k$.

III Flip:

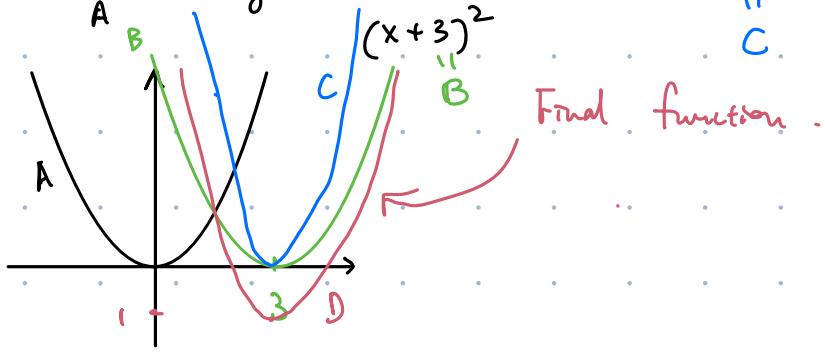
- $-f(x)$: vertical flip.
- $f(-x)$: horizontal flip.

- Shortcut:
 - If modification is out of parentheses, it is vertical: $f(x)+5, 5f(x)$
normal: up 5, larger 5.
 - If modification is inside parentheses, it is horizontal: $f(x+5), f(5x)$.
reverse: right 5, large $1/5$
left 5, small 5
- To understand the transformed function: decompose the function into simple pieces.

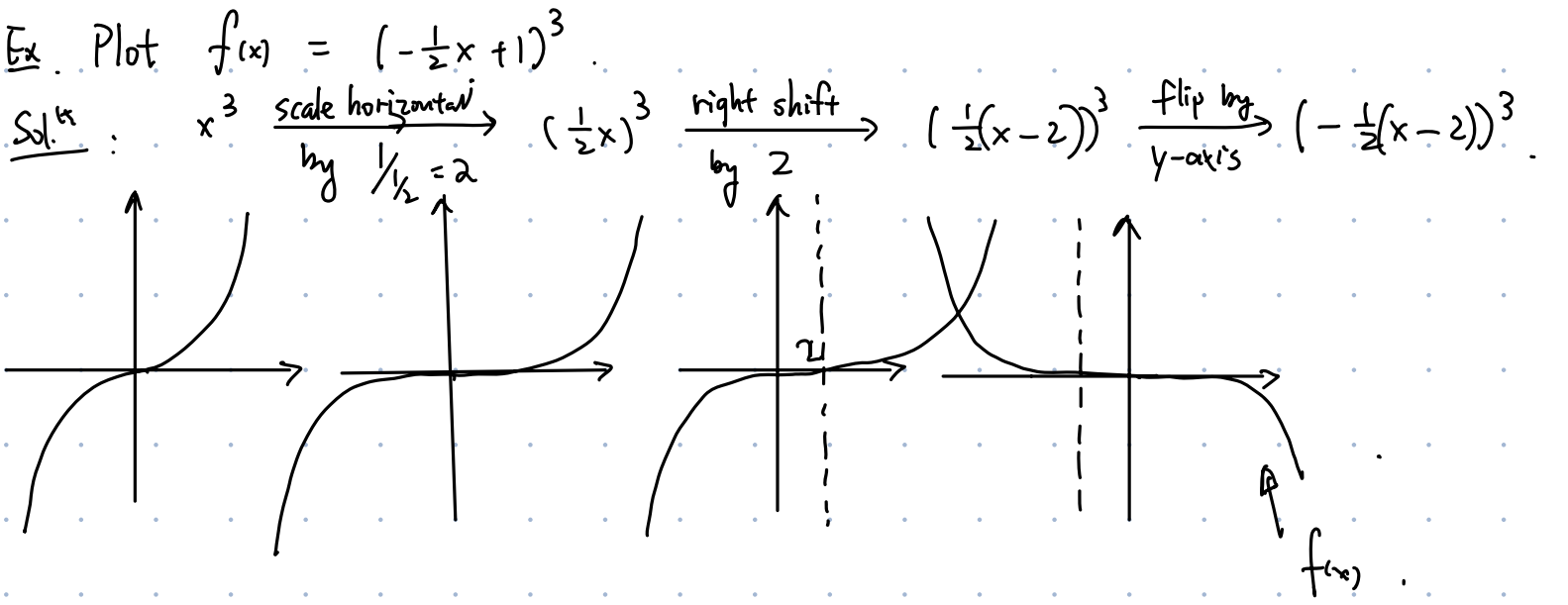
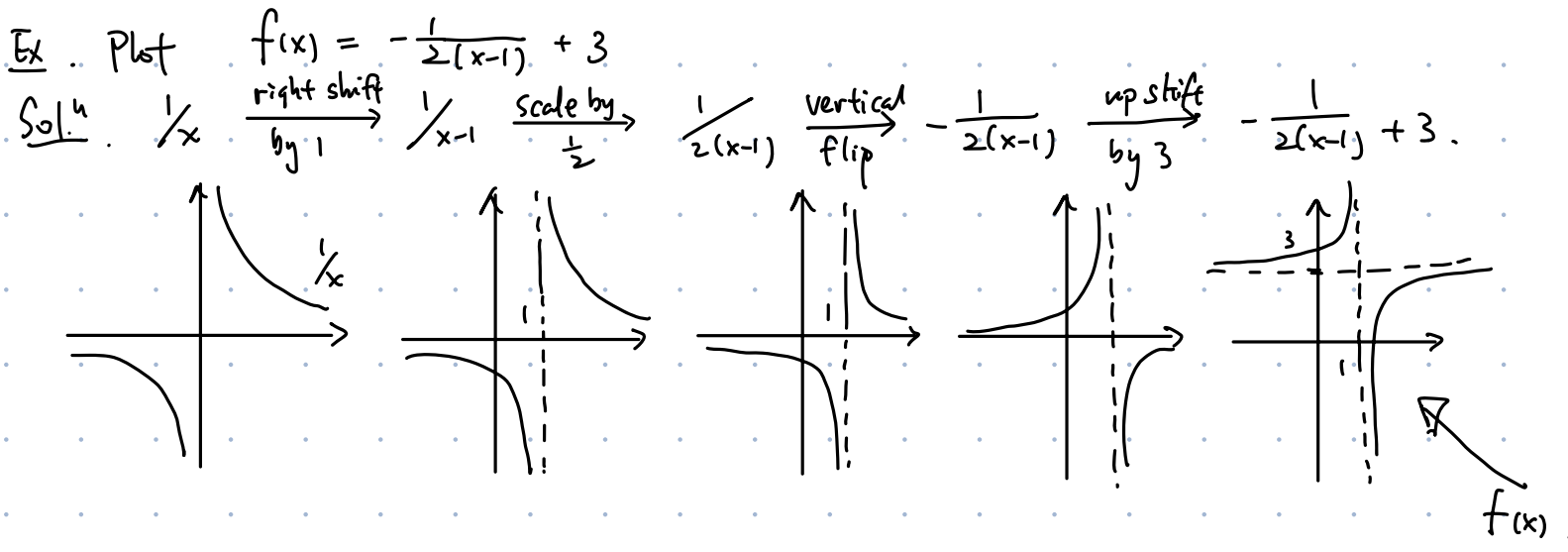
Ex. Plot $f(x) = 2(x+3)^2 - 1$.

Sol.ⁿ $x^2 \xrightarrow[\text{by } -3]{\text{right shift}} (x - (-3))^2 \xrightarrow[\text{by } 2]{\text{scale by}} 2(x+3)^2 \xrightarrow[\text{by } 1]{\text{shift down}} 2(x+3)^2 - 1$

A B C D

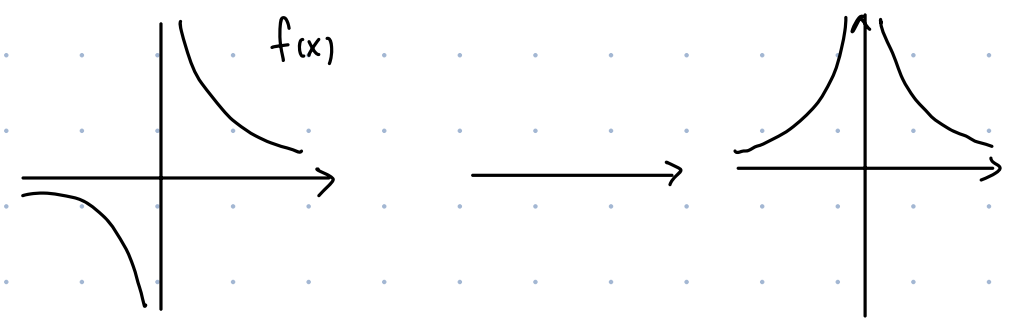


Final function.



More transforms:

④ Absolute value $|f(x)|$: mirror what's beneath x-axis up.



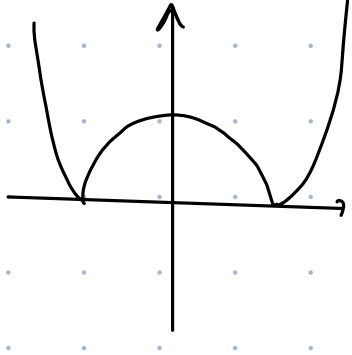
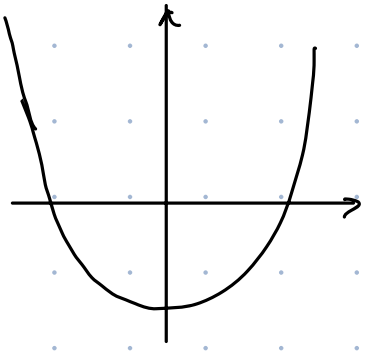
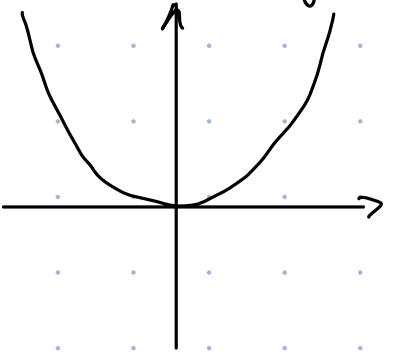
⑤ Reciprocals $\frac{1}{f(x)}$: keep the sign, but map tall to low / low to tall.



Ex. Plot

Sol.ⁿ

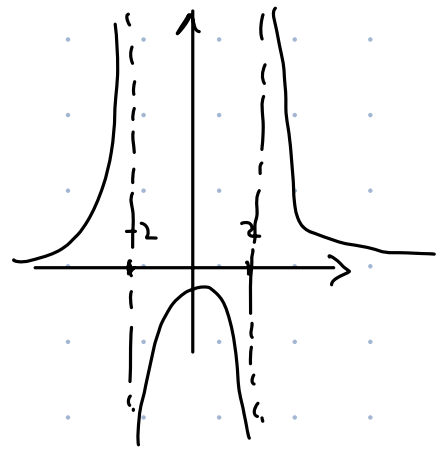
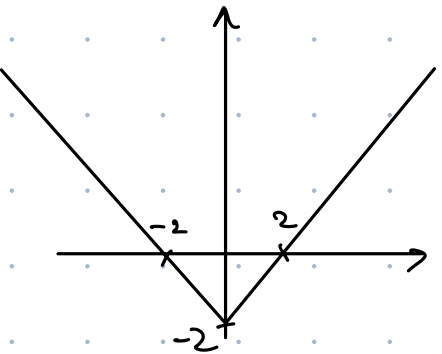
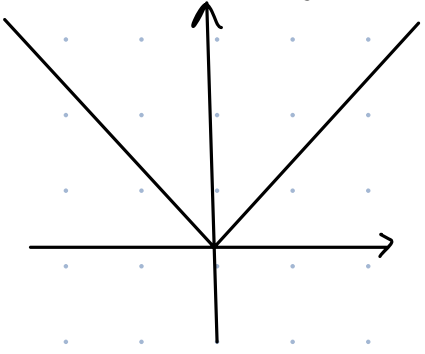
$$x^2 \xrightarrow[\text{by } 3]{\text{down shift}} x^2 - 3 \xrightarrow{|f|} |x^2 - 3|$$



Ex. Plot

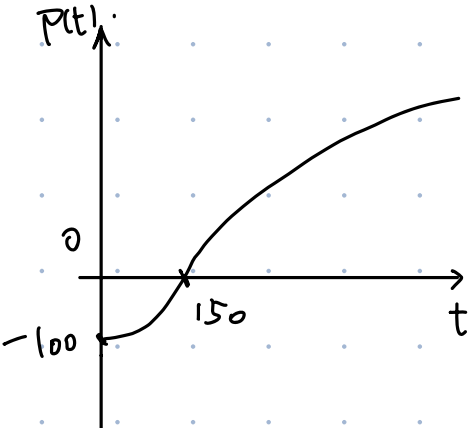
Sol.ⁿ

$$|x| \xrightarrow[\text{by } 2]{\text{down shift}} |x| - 2 \xrightarrow{1/f} \frac{1}{|x| - 2}$$



Ex.

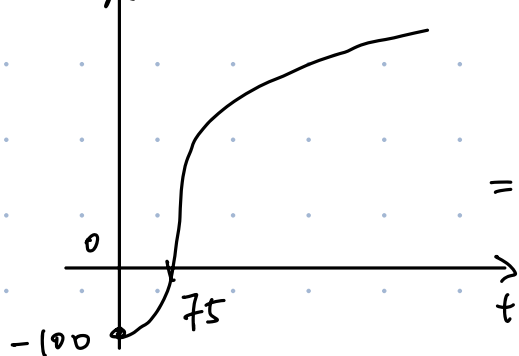
The profit function $P(t)$ of a company looks like, where $x =$ days of operation.



- Now they found a new method of production, that can produce twice as much each day. The cost of production per good does not change.

Q: Find the function of new Profit $P_1(t)$. Clearly label the intercepts on the axes.

Sol.ⁿ : $R_1(t)$



$R_1(t)$ = Profit of t days of production w/ new meth.
= Profit from goods produced w/ new method for t days

= Profit from 2x goods - - - - - old method for t days
= $R(2t)$.

• In the parentheses: horizontal, reverse ($2 \rightarrow \frac{1}{2}$)
Stretch by $\frac{1}{2}$.

- y-intercept = -100 : the company loses 100 if produced nothing.
- x-intercept = 75 : the company need to make 75 products to get positive profit.